TEACHING PHILOSOPHY

ADAM MARTENS

Historically, I tended to dislike teaching philosophies. As an undergraduate, I found them generally unhelpful—after all, I had to take the class regardless of the instructor's philosophy. Early in graduate school, I often dismissed them as mere bureaucratic requirements, useful only for checking a box on an application; a well-written essay on teaching seemed to serve administrators more than students or educators.

Over the past few years, however, I've transitioned from thinking of myself primarily as a student to embracing my role as an instructor. During my academic journey, I took 37 math classes and served as a TA or instructor for 15 semesters. This experience has given me ample time to reflect on what makes an effective math instructor at the university level and what kind of teacher I aspire to be. Through this transition and reflection, my opinion on teaching philosophies has changed. I now believe that, as a teacher, it's absolutely essential to have a clear understanding of what it is that you are exactly trying to achieve when leading a classroom through a semester.

So here I am, about to try to distill all that thought and reflection down down to just a few main points, which is of course completely unfeasible, but oh well...

1. Math is hard! But...

I bet I really blew your mind on that first one, eh? All those years of reflection about mathematics teaching, and my number one point that I can muster is a thought that quite possibly *every single person with a high school education* has had. Yes, of course math is hard! Despite this unanimous feeling, I feel that this point is not acknowledged enough when we discuss teaching (in particular, first-year calculus).

I often remind my first-year students that what they're learning is the culmination of a thousand years of mathematical advancement, capstoned (debatably) by the greatest scientist to ever live (also debatable), Isaac Newton. The calculus we teach today is even more rigorous and expansive than what Newton envisioned, yet we expect students to master it in just one or two semesters. It's natural to feel lost and confused when grappling with such material. The key is not whether you get lost, but whether you're willing to put in the work that is necessary to get *un*lost.

Learning mathematics is not a passive activity; it requires active engagement and persistent effort. Watching lectures or YouTube videos isn't enough (you would never learn to ride a bicycle by watching someone else do it). The way that students learn best is by working through complicated problems by themselves or in a small group. This is an uncomfortable process to learn how to do, and I would argue that too few students are adequately prepared for this by their high school experience. It is human nature to try to find an easy way out: apply a convenient formula, copy your friends work, look at the answer key, etc. But this process needs to be overcome (and quickly!) if a student wishes to succeed in mathematics.

My role as an instructor is to make students aware of this early in the course, to emphasize the importance of working through challenging problems by allowing class time for students to work on them with some guidance, and to encourage students to be open to making mistakes. In mathematics, it isn't about the mistakes you make, but rather about how *quickly* you can self-correct. You are going to fall off the bike a lot, so you need to learn how to land gracefully and get back on.

I didn't really grasp this approach until my third year of university. In a particularly challenging course on Rings and Modules, I found myself completely, and utterly, Toyota-Camry-in-a-six-foot-ditch kind of stuck. I relied on my friends for help on the assignments and was lost in lectures. About halfway through the semester, in desperation, I went to my professor's office hour, nearly in tears. I felt like there was no way I could pass this class. I don't know what I went to her office hour hoping to achieve, but the advice that she gave stunned me and certainly helped me get to where I am today. She said: (here is the resolution to the unresolved "But..." in the title)

..."You just have to do it"

Now that I am looking at those words on a page (or more accurately, a screen), it looks silly. But truly understanding what it means in the context of learning math takes some time. It means deliberately setting time aside *every day* to work on mathematics without looking at someone else's solutions, working problems out from scratch while showing all your work until you understand every implication thoroughly. When you're stuck on a problem, start by writing down everything you know (definitions, relating equations, possible techniques), and just ask yourself what the only possible thing that you can do is given the information in front of you. I still have to take this approach to learning; when reading a paper, for instance, I find that I don't *truly* understand it until I have written it out myself, line-by-line. You can't allow yourself to fall into a false sense of security by thinking something looks correct until you actually understand every implication. Eventually, you get comfortable enough with the material that such a rigorous process is no longer necessary, but it takes a while. Communicating this to my students, I believe, helps them to take ownership of their learning and to embrace the struggle as an essential part of their mathematical journey.

2. Learning math should be fun and memorable!

Since my first point was painfully obvious, I kind of figured I should go highly controversial on the second one. A large fraction of the cohort taking first year calculus is either ambivalent about mathematics or simply regards it as a necessary evil to get to the courses they actually *want* to take. I find this quite deplorable because I know from experience that learning math can be fun, engaging, and memorable while at the same time being *incredibly* difficult. When I think back to the math classes that I have enjoyed the most, which were also usually the same classes that I did the best in, they were the ones that I found most fun and exciting.

Injecting this fun and excitement into a first-year class (in which most of the students undoubtably have preconceived notions that math is dry and daunting), is, I find, necessary to retain attention and deepen engagement. Regarding *what* I speak about: I do my best to tell jokes, relay the history behind the subject, share personal anecdotes, and use relatable analogies and catchphrases that make abstract concepts stick. One of my favourite examples of the latter point is what my own first-year math professor would always say: "If it looks like a duck, walks like a duck, and quacks like a duck, then it's probably a duck!". He would say this so often that by the end of the term, none of us could think of ducks the same way again. But the point he was trying to make is so helpful when doing mathematics, which is: *Do the obvious*[1] *thing first.* So often, we (myself included) have a tendency to try to think through an entire problem before actually doing anything, which most of the time leads to overthinking. Instead, start by writing down what you know and precisely what you need to show, and 'build a bridge' between them one brick at a time.

I also believe in utilizing the tools available when teaching, whether that be props and visual aids, Desmos online graphing calculator, or even YouTube videos. I try in every class to do *something* like this to take the students eyes away from the blackboard and relieve the pressure of intently focusing, even if only for a couple minutes. Not only do these small visual respites help students grasp concepts, but I find they also help with general attentiveness.

I know from experience that, no matter how interesting the subject matter, a lecture that is delivered by someone standing in one place writing on a document camera the whole time can be very challenging to remain focused in. This 'standing in one place' is precisely the opposite way that I try to physically conduct myself. On the contrary, I move around the room a lot (sometimes speaking from the back of the class), which helps to decentralize peoples attention and keeps them on their toes. I take advantage of my booming voice, not all the time, of course, but at key 'AHA' moments during a lecture, with the hope that this helps to drive the main points home with the audience. I also try to speak and act with a lot of energy in the sense that I often move and speak quickly (at least, I speak quickly on the unimportant points!). I have often heard from students that my energy and passion for the subject are contagious and that my teaching style made the class very engaging and (for a few students at least) the "highlight of their week". Now I don't labour under the delusion that everyone is going to enjoy learning mathematics just because I am passionate about it, but I do think it helps.

3. The importance of inclusion

As an instructor, I'm an ambassador for the entire field of mathematics to my students. It's my responsibility to ensure that my classroom is inclusive, especially toward underrepresented minorities (the 'official' acronym I believe is URM). The lack of representation of URM in higher level mathematics is a complex social issue, and this isn't the place for discussing the causes, but rather to discuss *what can I do about it as an instructor?*

Beyond the obvious—avoiding discrimination—I believe in a more proactive approach. In a culture that can be unwelcoming, or even hostile at times to URM, instructors need to actively encourage these minorities. One way I do this is by deliberately encouraging wrong answers in class. This might sound counterintuitive for a math teacher, but I believe it helps destigmatize mistakes and encourages participation, especially from groups who may have otherwise not taken part because they may not have felt they belonged. Instead of simply telling a student that their answer is wrong, I ask them to explain their reasoning and praise the correct aspects. I also try to ask questions that have more than one correct answer and explain the nuances and subtleties between them. This creates a more welcoming environment and fosters a growth mindset.

[1] I recognize that 'obvious' is a dangerous word in mathematics, but I promise that I mean this in the most earnest way possible.

I also limit the number of times an individual student can answer questions in class to ensure that more voices are heard. When students work in small groups, I make an effort to ask students from URM to share their answers and redirect attention to them if they are interrupted. By continuously encouraging participation and ensuring that everyone's voice is heard, I hope to contribute to a more inclusive and supportive learning environment.

My motivation to continue to build an inclusive and welcoming learning environment has been reinforced by some positive feedback that I have received from my students, including, for example, that I have been "exceptionally welcoming and compassionate", "very empathetic with students who are struggling", that I have "helped to quell the notion [that] the general culture of mathematics [is] quite isolating", etc. Hearing these first-hand accounts that I have had a positive impact on my students' mathematics journey is the greatest motivation that I could ask for to continue to improve.

My biggest hope for **all** of the students is that, through my passion and excitement for this subject, I will inspire in them a lasting appreciation for mathematics, in spite of (or maybe even, because of) the fact that it may be one of the most challenging courses they will ever take.

Thank you for reading.